Reanalysis of the $(0^+, 1^+)$ states B_{s0} and B_{s1} with QCD sum rules

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Abstract

In this article, we calculate the masses and decay constants of the P-wave strange-bottomed mesons B_{s0} and B_{s1} with the QCD sum rules, and observe that the central values of the masses of the B_{s0} and B_{s1} are smaller than the corresponding BK and B^*K thresholds respectively, the strong decays $B_{s0} \to BK$ and $B_{s1} \to B^*K$ are kinematically forbidden. They can decay through the isospin violation precesses $B_{s0} \to B_s\eta \to B_s\pi^0$ and $B_{s1} \to B_s^*\eta \to B_s^*\pi^0$. The bottomed mesons B_{s0} and B_{s1}, just like their charmed cousins $D_{s0}(2317)$ and $D_{s1}(2460)$, maybe very narrow.

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Key words: B_{s0}, B_{s1}, QCD sum rules

1 Introduction

According to the heavy quark effective theory [1], in the limit $m_Q \to \infty$, the heavy quark decouples from the light degrees of freedom. $\vec{J} = \vec{S}_Q + \vec{j}_q$, $\vec{j}_q = \vec{S}_q + \vec{L}$, where $\vec{S}_Q$ and $\vec{S}_q$ are the spins of the heavy and light quarks respectively, $\vec{L}$ is the total angular momentum. For the P-wave strange-bottomed states, there are two degenerate doublets: the $j_q = \frac{1}{2}$ states B_{s0} and B_{s1}, and the $j_q = \frac{3}{2}$ states B_{s1}^* and B_{s2}^*. If kinematically allowed, the states with $j_q = \frac{1}{2}$ can decay via an S-wave transition, while the $j_q = \frac{3}{2}$ states undergo a D-wave transition; the decay widths of the states with $j_q = \frac{1}{2}$ are expected to be much broader than the corresponding $j_q = \frac{3}{2}$ states.

Recently, the CDF Collaboration reports the first observation of two narrow resonances consistent with the orbitally excited P-wave B_s mesons using 1 fb^{-1} of $p\overline{p}$ collisions at $\sqrt{s} = 1.96\,\mathrm{TeV}$ collected with the CDF II detector at the Fermilab Tevatron [2]. The masses are $M(B_{s1}^*) = (5829.4 \pm 0.7)\,\mathrm{MeV}$ and $M(B_{s2}^*) = (5839.7 \pm 0.7)\,\mathrm{MeV}$. The D0 Collaboration reports the direct observation of the excited P-wave state B_{s2}^* in fully reconstructed decays to B^+K^-, the mass is $(5839.6 \pm 1.1 \pm 0.7)\,\mathrm{MeV}$ [3]. The P-wave B_s states with spin-parity $J^P = (0^+, 1^+)$ are still lack experimental evidence, they maybe observed at the Tevatron or LHCb.

The masses of the B_s states with $J^P = (0^+, 1^+)$ have been estimated with the potential models, heavy quark effective theory and lattice QCD [4, 5, 6, 7, 8, 9, 10,

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Mesons	References
D, D_s, B, B_s	[15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27]
D^*, D_s^*, B^*, B_s^*	[14, 16, 21, 26, 27]
D_0, D_1, D_{s0}, D_{s1}	[26, 27]
$B_0, B_1(B_{s0}, B_{s1})$	[16, 21, 27]([16])

Table 1: The works on the charmed and bottomed heavy-light mesons with the QCD sum rules.

12, 13], they are listed in Table.2, from the table, we can see that the values are different from each other.

There have been many works on the charmed and bottomed heavy-light mesons with the QCD sum rules, they are classified roughly in Table.1. From the table, we can see that most of the works focus on the pseudoscalar mesons, the works on the scalar and axial-vector mesons are few.

In Ref.[16], Reinders, Yazaki and Rubinstein study the strange-bottomed $(0^+, 1^+)$ states (B_{s0}, B_{s1}) with the moment sum rules, due to the large threshold parameters $s_0 = 70\,\mathrm{GeV}^2$, they obtain large values $M_S = 6.29\,\mathrm{GeV}$ and $M_A = 6.34\,\mathrm{GeV}$, which are larger than the experimental values for the corresponding $(1^+, 2^+)$ states about $0.5\,\mathrm{GeV}$ [2, 3].

Although we can obtain useful predictions for the masses of the strange-bottomed $(0^+, 1^+)$ states (B_{s0}, B_{s1}) from the works of Narison with the symmetry properties [21, 27], it is valuable to study the strange-bottomed $(0^+, 1^+)$ states with the QCD sum rules directly.

In this article, we calculate the masses and decay constants of the B_{s0} and B_{s1} with the QCD sum rules [28, 29]. In the QCD sum rules, the operator product expansion is used to expand the time-ordered currents into a series of quark and gluon condensates which parameterize the long distance properties of the QCD vacuum. Based on current-hadron duality, we can obtain copious information about the hadronic parameters at the phenomenological side.

The article is arranged as follows: we derive the QCD sum rules for the masses and decay constants of the B_{s0} and B_{s1} in section 2; in section 3, numerical results and discussions; section 4 is reserved for conclusion.

2 QCD sum rules for the B_{s0} and B_{s1}

In the following, we write down the two-point correlation functions $\Pi_{\mu\nu}(p)$ and $\Pi_{SS}(p)$ in the QCD sum rules,

$$
\begin{aligned}
\Pi_{\mu\nu}(p) &= i\int d^4x e^{ip\cdot x}\langle 0|T\left\{J_\mu^A(x)J_\nu^{A\dagger}(0)\right\}|0\rangle\,, && (1)\\
\Pi_{SS}(p) &= i\int d^4x e^{ip\cdot x}\langle 0|T\left\{J_S(x)J_S^{\dagger}(0)\right\}|0\rangle\,, && (2)\\
J_\mu^A(x) &= \bar{s}(x)\gamma_\mu\gamma_5 b(x)\,,\\
J_S(x) &= \bar{s}(x)b(x)\,.
\end{aligned}
$$

The correlation function $\Pi_{\mu\nu}(p)$ can be decomposed as follows,

$$
\Pi_{\mu\nu}(p) = (-g_{\mu\nu}+\frac{p_\mu p_\nu}{p^2})\Pi_1(p^2)+p_\mu p_\nu\Pi_0(p^2)+\cdots \qquad (3)
$$

due to Lorentz covariance, we choose the tensor structure $-g_{\mu\nu}+\frac{p_\mu p_\nu}{p^2}$ for analysis.

After performing the standard procedure of the QCD sum rules, we obtain the following two sum rules,

$$
\begin{aligned}
f_S^2M_S^2e^{-\frac{M_S^2}{M^2}} &= \frac{3}{8\pi^2}\int_{m_b^2}^{s_S^0}ds e^{-\frac{s}{M^2}}s\left(1-\frac{m_b^2}{s}\right)^2\left[1-\frac{2m_bm_s}{s-m_b^2}+\frac{4}{3}\frac{\alpha_s(s)}{\pi}R_0(\frac{m_b^2}{s})\right]\\
&+e^{-\frac{m_b^2}{M^2}}\left[m_b\langle\bar{s}s\rangle+\frac{m_s\langle\bar{s}s\rangle}{2}\left(1+\frac{m_b^2}{M^2}\right)+\frac{1}{12}\langle\frac{\alpha_sGG}{\pi}\rangle\right.\\
&\left.+\frac{m_b\langle\bar{s}g_s\sigma Gs\rangle}{2M^2}\left(1-\frac{m_b^2}{2M^2}\right)-\frac{16\pi\alpha_s\langle\bar{s}s\rangle^2}{27M^2}\left(1-\frac{m_b^2}{4M^2}-\frac{m_b^4}{12M^4}\right)\right]\,,
\end{aligned}
\qquad (4)
$$

$$
\begin{aligned}
f_A^2M_A^2e^{-\frac{M_A^2}{M^2}} &= \frac{1}{8\pi^2}\int_{m_b^2}^{s_A^0}ds e^{-\frac{s}{M^2}}s\left(1-\frac{m_b^2}{s}\right)^2\left(2+\frac{m_b^2}{s}\right)\left[1-\frac{3m_bm_ss}{(2s+m_b^2)(s-m_b^2)}\right.\\
&\left.+\frac{4}{3}\frac{\alpha_s(s)}{\pi}R_1(\frac{m_b^2}{s})\right]+e^{-\frac{m_b^2}{M^2}}\left[m_b\langle\bar{s}s\rangle+\frac{m_b^2m_s\langle\bar{s}s\rangle}{2M^2}-\frac{1}{12}\langle\frac{\alpha_sGG}{\pi}\rangle\right.\\
&\left.-\frac{m_b^3\langle\bar{s}g_s\sigma Gs\rangle}{4M^4}+\frac{32\pi\alpha_s\langle\bar{s}s\rangle^2}{81M^2}\left(1+\frac{m_b^2}{M^2}-\frac{m_b^4}{8M^4}\right)\right]\,,
\end{aligned}
\qquad (5)
$$

where

$$
\begin{aligned}
R_0(x) &= \frac{9}{4} + 2\mathrm{Li}_2(x) + \ln x \ln(1-x) - \frac{3}{2}\ln\frac{1-x}{x} - \ln(1-x) + x\ln\frac{1-x}{x} \\
&\quad -\frac{x}{1-x}\ln x\,, \qquad (6) \\
R_1(x) &= \frac{13}{4} + 2\mathrm{Li}_2(x) + \ln x \ln(1-x) - \frac{3}{2}\ln\frac{1-x}{x} - \ln(1-x) + x\ln\frac{1-x}{x} \\
&\quad -\frac{x}{1-x}\ln x + \frac{(3+x)(1-x)}{2+x}\ln\frac{1-x}{x} - \frac{2x}{(2+x)(1-x)^2}\ln x \\
&\quad -\frac{5}{2+x} - \frac{2x}{2+x} - \frac{2x}{(2+x)(1-x)}\,, \qquad (7)
\end{aligned}
$$

$\mathrm{Li}_2(x) = -\int_0^x dt\, t^{-1}\ln(1-t)$, $GG = G_{\mu\nu}G^{\mu\nu}$, $\sigma G = \sigma_{\mu\nu}G^{\mu\nu}$, $\frac{\alpha_s(s)}{4\pi} = \frac{1}{\frac{23}{3}\log(s/\Lambda_{\mathrm{QCD}}^2)}$, $\Lambda_{QCD} = 226\mathrm{MeV}$ and $\alpha_s(1\mathrm{GeV}) = 0.514$ [30]. We have used the standard definitions for the decay constants f_S and f_A,

$$
\begin{aligned}
f_S M_S &= \langle 0|J_S(0)|B_{s0}(p)\rangle\,, \\
f_A M_A \epsilon_\mu &= \langle 0|J_\mu^A(0)|B_{s1}(p)\rangle\,. \qquad (8)
\end{aligned}
$$

The spectral densities of the heavy-light $(0^+,1^+)$ mesons can be obtained from the corresponding $(0^-,1^-)$ mesons with the simple replacement $m_Q \to -m_Q$, where the m_Q stands for the masses of the heavy quarks. Although the expressions from different references have minor differences from each other, the contributions from those terms are usually of minor importance. We have consulted the analytical expressions presented in Refs.[14, 15, 16, 17, 27, 31, 32, 33] to obtain our main results.

Differentiating the Eqs.(4-5) with respect to $\frac{1}{M^2}$, then eliminate the quantities f_S and f_A, we can obtain two sum rules for the masses of the B_{s0} and B_{s1}, respectively.

3 Numerical results and discussions

The input parameters are taken to be the standard values $\langle\bar{q}q\rangle = -(0.24\pm0.01\,\mathrm{GeV})^3$, $\langle\bar{s}s\rangle = (0.8\pm0.2)\langle\bar{q}q\rangle$, $\langle\bar{s}g_s\sigma Gs\rangle = m_0^2\langle\bar{s}s\rangle$, $m_0^2 = (0.8\pm0.2)\,\mathrm{GeV}^2$, $\langle\frac{\alpha_s GG}{\pi}\rangle = (0.33\,\mathrm{GeV})^4$, $m_s = (0.14\pm0.01)\,\mathrm{GeV}$ and $m_b = (4.7\pm0.1)\,\mathrm{GeV}$ at the energy scale about $\mu = 1\,\mathrm{GeV}$ [28, 29, 34].

In Refs.[26, 27], different predictions for the masses of the scalar mesons D_{s0} and D_0 are obtained due to different values of the threshold parameters s_0 and charmed quark mass m_c. The mass m_b is very large comparing with the m_c, the uncertainty about 0.1 GeV cannot change the predictions remarkably. We choose the threshold parameters s_0 with the guide of the experimental data and predictions of the potential models to reduce the uncertainty.

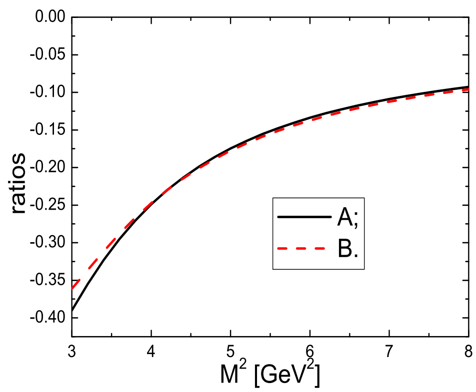

Figure 1: The ratios between the contributions of the non-perturbative terms and the perturbative terms with variation of the Borel parameter M^2, A for the B_{s0} and B for the B_{s1}.

In 2006, the BaBar Collaboration observed a new $c\bar{s}$ state $D_s(2860)$ with the mass $M = (2856.6 \pm 1.5 \pm 5.0)\,\text{MeV}$, width $\Gamma = (48 \pm 7 \pm 10)\,\text{MeV}$ and possible spin-parity 0^+, 1^-, 2^+, $\cdots$ [35]. It has been interpreted as the first radial excitation of the 0^+ state $D_{s0}(2317)$ in Refs.[36, 37], although other identifications are not excluded. The energy gap between the $2P$ and $1P$ scalar $c\bar{s}$ states is about $\delta M_S = 0.539\,\text{GeV}$. In 2007, the Belle Collaboration observed a new resonance $D_s(2700)$ in the decay $B^+ \to \bar{D}^0 D_s(2700) \to \bar{D}^0 D^0 K^+$, which has the mass $M_V = 2708 \pm 9^{+11}_{-10}\,\text{MeV}$, width $\Gamma_V = 108 \pm 23^{+36}_{-31}\,\text{MeV}$, and spin-parity 1^-[38]. They interpret the $D_s(2700)$ as a $c\bar{s}$ meson, the potential models predict a radially excited 2^3S_1 ($c\bar{s}$) state with a mass about $(2710 - 2720)\,\text{MeV}$ [39, 40], so the energy gap between the $2S$ and $1S$ vector $c\bar{s}$ states is about $\delta M_V = 0.596\,\text{GeV}$.

If the masses of the P-wave strange-bottomed mesons are of the same order (about $5.8\,\text{GeV}$ [2, 3]) and the energy gap between the ground state and the first radially excited state is about $0.5\,\text{GeV}$ (just like the $c\bar{s}$ mesons), we can make a rough estimation for the masses of the first radially excited $(0^+, 1^+)$ strange-bottomed states, $M_r \approx (5.8 + 0.5)\,\text{GeV}$. The threshold parameters should be chosen as $s_0 < M_r^2 \approx 40\,\text{GeV}^2$, which are consistent with the predictions of potential models [9].

The threshold parameters can be taken as $s_S^0 = (37 \pm 1)\,\text{GeV}^2$ and $s_A^0 = (38 \pm 1)\,\text{GeV}^2$, which are below the corresponding masses of the first radially excited states, $M_{Sr} = 6.264\,\text{GeV}$ for the B_{s0} and $M_{Ar} = 6.296\,\text{GeV}$ for the B_{s1} in the potential model, see Ref.[9]. The threshold parameters $s_0 = 70\,\text{GeV}^2$ chosen by Reinders, Yazaki and Rubinstein in Ref.[16] are too large to make reliable predictions.

The Borel parameters are taken as $M^2 = (4{-}6)\,\text{GeV}^2$ for the B_{s0} (see Eq.(4)) and $M^2 = (5 - 7)\,\text{GeV}^2$ for the B_{s1} (see Eq.(5)). In those regions, the contributions from the pole terms are larger than 50%; furthermore, the dominating contributions come from the perturbative terms. In Fig.1, we plot the ratios between the contributions

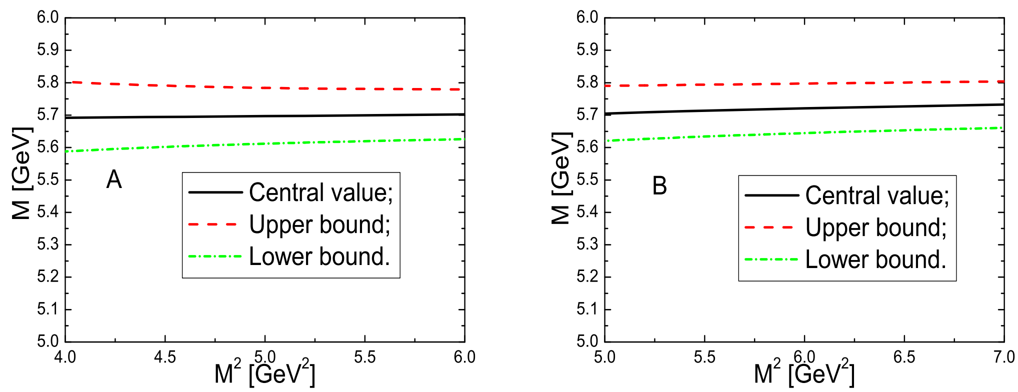

Figure 2: $M_S(A)$ and $M_A(B)$ with variation of the Borel parameter M^2.

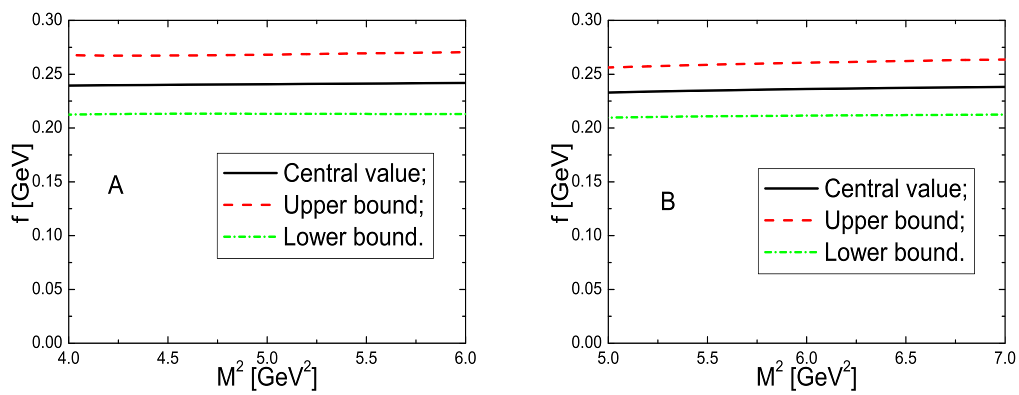

Figure 3: $f_S(A)$ and $f_A(B)$ with variation of the Borel parameter M^2.

of the non-perturbative terms (come from the vacuum condensate $\langle\bar{s}s\rangle$ mainly) and the perturbative terms with variation of the Borel parameter M^2. In the region $M^2 = (4-7)\,\text{GeV}^2$, the ratios are about $-(0.10-0.25)$. The criterions (pole dominance and convergence of the operator product expansion) of the QCD sum rules are well satisfied, our predictions are robust.

Taking into account all uncertainties of the input parameters, finally we obtain the values of the masses and decay constants of the P-wave heavy mesons B_{s0} and B_{s1}, which are shown in Figs.(2-3), respectively,

$$\begin{aligned}
M_S &= (5.70 \pm 0.11)\,\text{GeV}\,, \\
M_A &= (5.72 \pm 0.09)\,\text{GeV}\,, \\
f_S &= (0.24 \pm 0.03)\,\text{GeV}\,, \\
f_A &= (0.24 \pm 0.02)\,\text{GeV}\,. \qquad (9)
\end{aligned}$$

	M_S(GeV)	M_A(GeV)
[4]	5.841	5.831
[5]	5.830	5.786
[6]	5.718	5.765
[7]	5.71	5.77
[8]	5.756 ± 0.031	5.804 ± 0.031
[9]	5.804	5.842
[10]	5.679	5.713
[11]	5.689	5.734
This work	5.70 ± 0.11	5.72 ± 0.09

Table 2: Theoretical estimations for the masses of the B_{s0} and B_{s1} from different models.

From the experimental data [41], $m_B = 5.279\,\text{GeV}$, $m_K = 0.495\,\text{GeV}$ and $m_{B^*} = 5.325\,\text{GeV}$, we can see that the central values are below the corresponding BK and B^*K thresholds respectively, $m_{BK} = 5.774\,\text{GeV}$ and $m_{B^*K} = 5.820\,\text{GeV}$. The strong decays $B_{s0} \to BK$ and $B_{s1} \to B^*K$ are kinematically forbidden, the P-wave heavy mesons B_{s0} and B_{s1} can decay through the isospin violation precesses $B_{s0} \to B_s\eta \to B_s\pi^0$ and $B_{s1} \to B_s^*\eta \to B_s^*\pi^0$ respectively. The η and π^0 transition matrix is very small according to Dashen's theorem [42],

$$t_{\eta\pi} = \langle \pi^0 | \mathcal{H} | \eta \rangle = -0.003\,\text{GeV}^2\,, \tag{10}$$

the P-wave bottomed mesons B_{s0} and B_{s1}, just like their charmed cousins $D_{s0}(2317)$ and $D_{s1}(2460)$, maybe very narrow [43, 44].

4 Conclusion

In this article, we calculate the masses and decay constants of the P-wave strange-bottomed mesons B_{s0} and B_{s1} with the QCD sum rules, and observe that the central values of the masses are smaller than the corresponding BK and B^*K thresholds respectively, the strong decays $B_{s0} \to BK$ and $B_{s1} \to B^*K$ are kinematically forbidden. They can decay through the isospin violation precesses $B_{s0} \to B_s\eta \to B_s\pi^0$ and $B_{s1} \to B_s^*\eta \to B_s^*\pi^0$. The bottomed mesons B_{s0} and B_{s1}, just like their charmed cousins $D_{s0}(2317)$ and $D_{s1}(2460)$, maybe very narrow.

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